Modal MRTD Approaches for the Efficient Analysis of Waveguide Discontinuities

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***Abstract*—In this paper, the multi-resolution time-domain (MRTD) technique is applied to the waveguide discontinuity problem for the fast-scattering parameter computation. To improve the computational efficiency, both three-dimensional (3-D) waveguide region, including discontinuities, and one dimensional (1-D) homogeneous waveguide region, terminated with the modal absorbing boundary condition (ABC), are simulated in the wavelet domain with the mode composition/expansion algorithm from the modal analysis. A WG-90 rectangular waveguide with a thick asymmetric iris is analyzed and the numerical results are compared with conventional finite-difference time-domain (FDTD) results and mode-matching results.**

***Index Terms*—FDTD method, modal analysis, MRTD method, waveguide.**

I. INTRODUCTION

THE finite difference time-domain (FDTD) method provides a relatively simple way of modeling various complex structures and has been one of the most powerful tools for the analysis of various applied electromagnetic problems [1]. Recently, to improve the computational efficiency, the multiresolution theory has been applied to the FDTD method and leads to multi-resolution time domain (MRTD) techniques [2]–[4]. The MRTD approaches have been applied to analyze various electromagnetic structures with success [5] and also employed in the analysis of waveguide discontinuity problems [6]–[8] to obtain savings in memory or computation time.

For the scattering parameter computation of waveguide discontinuity problems with the conventional unimodal absorbing boundary condition (ABC), considerable numerical costs are required because one simulates a long uniform auxiliary waveguide of input–output port in order to extract the dominant waveguide mode. However, this difficulty can be overcome by simulating one-dimensional (1-D) mode wave equation and imposing appropriate boundary conditions on each waveguide mode at the input–output ports [9]–[11].

In this paper, the MRTD technique is applied to the waveguide discontinuity problem for the fast scattering parameter computation. To improve the computational efficiency and get the advantages of both MRTD and modal analysis, modal absorption techniques are implemented in the wavelet domain with the Haar-wavelet MRTD scheme [3].

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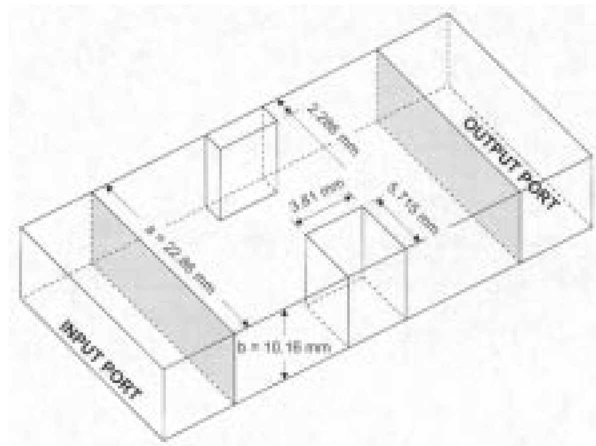

Fig. 1. WG-90 rectangular waveguide with a thick asymmetric iris.

II. THEORETICAL BACKGROUND AND FORMULATION

Fig. 1 shows the geometry of a WG-90 rectangular waveguide with a thick asymmetric iris. In the conventional FDTD simulation with the unimodal ABC, the uniform auxiliary waveguide of the input–output ports is introduced to attenuate the evanescent waveguide modes and extract the dominant waveguide mode, which requires additional numerical costs for the analysis of waveguide discontinuities (3-D FDTD problems). This difficulty can be solved by applying modal analysis techniques, such as the mode composition/expansion algorithm and modal ABCs, based on the orthogonality of modes in a hollow waveguide (1-D mode FDTD problems). Moreover, MRTD gives us the opportunity to further improve numerical efficiencies by reducing computation time or memory storage. Therefore, the proposed scheme employs multiresolution theory in both 3-D waveguide region of discontinuities (3-D MRTD problems) and 1-D homogeneous waveguide region of the input–output ports (1-D mode MRTD problems) as shown in Fig. 2.

Three–dimensional (3-D) MRTD difference equations and 1-D mode MRTD difference equations are obtained by discretizing the differential form of the Maxwell's equations and the mode wave equation in the one-scaling-level Haar basis and using the Galerkin's procedure, respectively [3]. Interfacing 3-D MRTD meshes and 1-D MRTD nodes is needed in the proposed scheme as the FDTD formulations, and there are two options to convert 1-D formulation to 3-D formulation and vice versa. The first choice is to treat the interface between

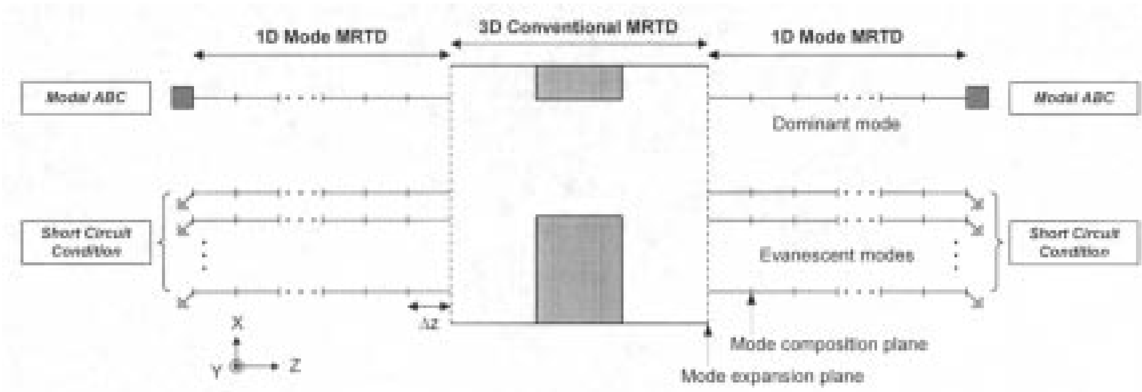

Fig. 2. Proposed algorithm to improve the computational efficiency of the FDTD simulations.

1-D and 3-D MRTD formulations in the physical domain, which employs the same equations as the FDTD formulation (MRTD–FDTD–MRTD) [9]–[11]. In the parlance of the MRTD formulation, these equations can be rewritten as follows

$$ {}^{n}_{ijk}E_t^{\phi\phi\phi} = \sum_m {}^{n}_{k}V_m^{\phi} \cdot {}_{ij}e_m^{\phi\phi} \tag{1a} $$

$$ {}^{n}_{k}V_m^{\phi} = \frac{\sum_i \sum_j {}^{n}_{ijk}E_t^{\phi\phi\phi} \cdot {}_{ij}e_m^{\phi\phi}}{\sum_i \sum_j |{}_{ij}e_m^{\phi\phi}|^2} \tag{1b} $$

where E_t, V_m, and e_m are the wavelet coefficients of the transverse electric field, the modal amplitude of the order m, and the transverse electric field pattern of the mth order mode, respectively. The integer i, j, and k indicate that the corresponding basis function is located at $x = i\Delta x$, $y = j\Delta y$, and $z = k\Delta z$ in the spatial lattice. The index n denotes temporal grid point $t = n\Delta t$. (1) assumes uniform spatial and temporal grid, and the summation in (1b) is done over the waveguide cross section. Conversion of $\vec{E}_t$ to V_m (V_m to $\vec{E}_t$) is done by mode expansion (mode composition) of the transverse electric field (the modal amplitude) obtained from the basis transformation relation between the physical domain field values and the wavelet domain field coefficients on the mode expansion plane (the mode composition plane) [3]. Then, we can transform the physical domain modal amplitude into the wavelet domain modal coefficients and apply appropriate boundary conditions (the modal ABC for the dominant mode and the short circuit condition for the higher order modes) to these coeffiecients.

The second choice treats the interface between 1-D and 3-D MRTD formulations in the wavelet domain, which uses the wavelet domain mode composition/expansion equations as follows (MRTD–MRTD–MRTD) in (2a) and (2b), shown at the bottom of the page, where the basis functions $\xi = \phi\phi$, $\phi\psi$, $\psi\phi$, $\psi\psi$ and $\zeta = \phi$, ψ are along the cross section of the waveguide (x–y plane) and the propagation direction (z-coordinate), respectively. The wavelet coefficients of the transverse electric field pattern of the mth order mode e_m^{ξ} can be obtained by the inner product between the transverse electric field pattern and the corresponding basis functions. The main distinction between this and the former scheme is the location of the mode composition/expansion plane. In the MRTD–MRTD–MRTD scheme, the distance between mode composition plane and mode expansion plane is two cells long in space, which is double the length of the former scheme. Conversion of $\vec{E}_t$ to V_m (V_m to $\vec{E}_t$) is done in the wavelet domain by the wavelet domain mode expansion (2b) [the wavelet domain mode composition (2a)]. Then, we can apply the wavelet domain modal ABC to the dominant modal coefficient and the short circuit condition to the higher order modal coefficients.

III. Numerical Results

The WG-90 rectangular waveguide with a thick asymmetric iris shown in Fig. 1 is analyzed with the modal PML technique [12] via FDTD simulations and the proposed schemes. In the proposed schemes, the modal PML for the dominant waveguide mode and the edge condition for singularities near an asymmetric iris [13] are implemented with the Haar-wavelet MRTD scheme. In the fine grid FDTD simulation, the space step sizes are $\Delta x = 0.381$ mm, $\Delta y = 1.016$ mm, and $\Delta z = 1$ mm. The time step size is $\Delta t \approx 0.95$ ps and the number of time iterations is 4000. In MRTD simulations, the space step sizes are $\Delta x = 0.762$ mm, $\Delta y = 2.032$ mm, and $\Delta z = 2$ mm. The

$$ {}^{n}_{ijk}E_t^{\xi\zeta} = \sum_m {}^{n}_{k}V_m^{\zeta} \cdot {}_{ij}e_m^{\xi} \tag{2a} $$

$$ {}^{n}_{k}V_m^{\zeta} = \frac{\sum_i \sum_j \left({}^{n}_{ijk}E_t^{\phi\phi\zeta} \cdot {}_{ij}e_m^{\phi\phi} + {}^{n}_{ijk}E_t^{\psi\phi\zeta} \cdot {}_{ij}e_m^{\psi\phi} + {}^{n}_{ijk}E_t^{\phi\psi\zeta} \cdot {}_{ij}e_m^{\phi\psi} + {}^{n}_{ijk}E_t^{\psi\psi\zeta} \cdot {}_{ij}e_m^{\psi\psi} \right)}{\sum_i \sum_j \left(|{}_{ij}e_m^{\phi\phi}|^2 + |{}_{ij}e_m^{\psi\phi}|^2 + |{}_{ij}e_m^{\phi\psi}|^2 + |{}_{ij}e_m^{\psi\psi}|^2 \right)} \tag{2b} $$

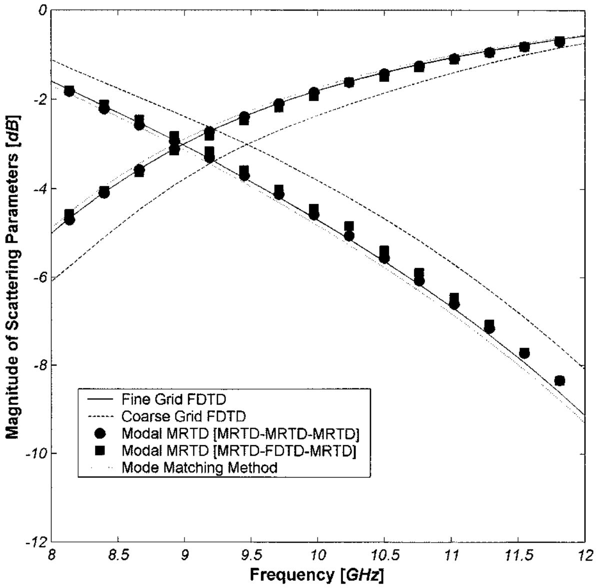

Fig. 3. Scattering parameters with the different simulation methods.

TABLE I
COMPARISON OF THE COMPUTATION TIME OF THE FINE GRID FDTD AND THE PROPOSED METHODS

	Fine grid FDTD	Modal MRTD approaches	
		MRTD-MRTD-MRTD	MRTD-FDTD-MRTD
Simulation Time	107.6 sec	61.07 sec	64.87 sec

time step size is $\Delta t \approx 1.9$ ps and the number of time iterations is 2000. Analysis conditions of the coarse grid FDTD are the same as those of MRTD simulations. All simulations were performed on a Pentium-III PC machine (650 MHz CPU and 128 Mbytes RAM).

Fig. 3 shows scattering parameters of the waveguide with different methods. The results of the fine grid FDTD, mode matching method, and the proposed schemes show better agreement than the coarse grid FDTD. Table I shows the computation time of the fine grid FDTD and the proposed schemes. The results show that the proposed approaches provide comparable numerical accuracy for structure of interest with the reduced simulation time.

IV. CONCLUSION

We have approached the problem of scattering parameter computation of waveguide structures by combining the Haar-wavelet MRTD scheme with the mode composition/expansion algorithm and the modal ABC based on the orthogonality of modes in a hollow waveguide. Numerical results demonstrated that the computational efficiency is further improved by the proposed schemes.

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